

**ALTERNATIVE MATHEMATICAL PROGRAMMING FORMULATIONS
FOR FSS SYNTHESIS**

by

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16. Abstract (Limit: 200 words) In this report, a variety of mathematical programming models, and two solution strategies, are suggested for the problem of allocating orbital positions to (synthesizing) satellites in the Fixed Satellite Service. Mixed integer programming and almost linear programming formulations are presented in detail for each of two objectives: (1) positioning satellites as closely as possible to specified "desired" locations, and (2) minimizing the total length of the geostationary arc allocated to the satellites whose positions are to be determined. Computational results for mixed integer and almost linear programming models, with the objective of positioning satellites as closely as possible to their desired locations, are reported for three six-administration test problems and a thirteen-administration test problem.			
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INTRODUCTION

In this report, we consider the Fixed Satellite Service (FSS) system synthesis problem, which can be described as follows: Communications satellites are to be positioned in the geostationary orbit and are to be assigned frequencies for transmitting signals to their intended service areas. The primary goal is to assign locations and frequencies to the satellites so that interference does not exceed a specified acceptable level. Other possible goals or objectives are: conservation of the geostationary orbit via minimization of the orbital arc occupied by the satellites in question, or positioning the satellites as closely as possible to specified "desired" locations.

Our primary purpose is to suggest alternative mathematical programming formulations of satellite system synthesis problems in this report. We present in detail mixed integer programming and almost linear programming formulations for each of two objectives: (1) positioning satellites as closely as possible to specified "desired" locations, and (2) minimizing the total length of the geostationary arc allocated to the satellites to be positioned. We report computational results for four test problems with this first objective. We also list additional possible objectives and review other satellite system synthesis models. Any of the models suggested in this paper can be modified to accommodate pre-existing satellite systems.

Space communications, and particularly FSS allotments, are to be addressed at the World Administrative Radio Conference to be held in 1988 (WARC-88). We think that optimization models, such as the ones we suggest herein, will be aids in the complex decision-making process that lies ahead for the WARC-88 delegates.

LITERATURE REVIEW

Mathematical programming formulations of satellite system synthesis problems have already received attention in the literature. Some of the approaches we discuss were originally intended for Broadcasting Satellite Service (BSS) system synthesis. We think that these approaches, with limited modifications, are applicable to FSS system synthesis problems as well.

Many approaches suggested have considered only the frequency aspect of the problem. For example, Cameron proposed an integer programming formulation that assigns one channel to each service area by solving a sequence of set covering problems [2]. The approach in this model is to conserve the spectrum by minimizing the number of channels needed, while enforcing constraints on co-channel interference. Levis, Martin, Wang, and Gonsalvez [8] present two integer programming formulations of the same frequency assignment problem. They also suggest an integer programming formulation that considers the assignment of multiple channels to a single service area

and takes into account adjacent-channel interference. The objective in their formulation is to minimize the bandwidth utilized. Baybars [1] has also suggested an integer programming model that seeks to minimize the number of channels used while considering both co-channel and adjacent-channel interference.

Ito, Mizuno, and Muratani have formulated a satellite system synthesis model that considers the assignment of only satellite locations [7]. Their model is a nonlinear program, which they suggest solving via the sequential unconstrained minimization technique. The objective is to minimize the total length of the orbital arc allocated to the satellites to be positioned; restrictions on single-entry and aggregate inter-system interference are enforced. Their model is "evolutional": a launch sequence of the satellites to be positioned is specified a priori; satellites are then added according to the assumed launch sequence. The problem is easier to solve because of its evolutionary nature; however, completed solutions may be suboptimal.

A nonlinear programming formulation that seeks to specify the assignment of both locations and frequencies has been suggested by Levis, Martin, Gonsalvez, and Wang [9] for the problem of synthesizing satellite systems in the Broadcasting Satellite Service (BSS). This model is formulated with the intention of maximizing the minimum aggregate carrier-to-interference (C/I) ratio over all given test points for the down-link. Assigned locations and frequencies are restricted to being within specified bounds. Levis, Martin,

Gonsalvez, and Wang [9] and Martin et al. [11] recommend solving this model with an extended gradient search procedure. Reilly, Levis, et al. [15] have implemented a cyclic coordinate search procedure to solve this same model. At best, these search procedures are heuristic methods for solving synthesis problems. Reilly, Mount-Campbell, et al. [16] describe an extensive experiment conducted to assess the performance of these search methods on a small test problem. Their findings indicate that the cyclic coordinate method consistently finds better synthesis solutions at the expense of greater computing time.

Other heuristic procedures have been suggested for satellite system synthesis problems. For example, Chouinard and Vachon describe an enumerative method for making channel and polarization assignments, given fixed orbital positions for the satellites being considered [3]. Nedzela and Sidney have developed two algorithms for assigning locations, channels, and polarizations to satellite systems [12]. In these algorithms, one satellite is selected for location, channel, and polarization assignments at a time. This satellite is selected because it has the least remaining "freedom" for assignments among the satellites yet to be selected. An interactive method for assigning locations, channels, and polarizations is described by Christensen [4].

Ottey, Sullivan, and Zusman [13] suggest several objectives related to interference for mathematical programming formulations of satellite synthesis problems.

The synthesis models we present in this report differ from others we have mentioned in that we make no assumptions about a launch sequence or about fixed orbital positions as other authors have done. We do not equate topocentric and geocentric angles as Ito, Mizuno, and Muratani did [7]. These new models are extensions of models suggested by Reilly, Levis, et al. [15], Levis, Wang, et al. [10], and Wang [18]. Finally, we also suggest five possible objectives that we think have not appeared in the open literature before.

MINIMUM SATELLITE SEPARATIONS

The formulations described below rely on the existence of a known required minimum separation, measured in degrees of geostationary orbital arc, for each pair of satellites. As far as the models and solution methods discussed here are concerned, these separation values might be selected arbitrarily, e.g., a uniform 2° separation or separations fixed by international agreement; however, values that relate more directly to the achievement of acceptable inter-system interference can be incorporated as well.

In our experimental work, we have used separations calculated with a computerized procedure developed by Wang [18] for determining the required minimum orbital separation between two satellites, with elliptical-beam antennas, needed to assure that single-entry co-channel C/I ratios at assumed ground stations (test points) along

the boundaries of the areas served by the satellites are at least equal to some threshold, for example, 30 dB. This required separation varies as the mean position of the satellites changes, but the variation appears to be small for at least some practical scenarios. Wang therefore suggests that all feasible orbital locations be considered when calculating the required minimum separation values, and that the maximum of these for each satellite pair over the allowable range of orbit positions be used in satellite system synthesis models. This was done in the examples shown later in this report. Interested readers are directed to Wang [18], Levis, Wang, et al. [10], and Reilly, Levis, et al. [15] for a more complete treatment of this separation concept; also see Yamamura and Levis [19] for similar work regarding satellites with circular-beam antennas.

Other models for satellite system synthesis have used similar separation concepts. For example, an extension of the model developed by Ito, Mizuno, and Muratani [7], which is described in [17], uses a spacing matrix of satellite separation values calculated to guarantee that single-entry interference requirements will be met. The spacing values are computed, given the current locations of the satellites already positioned. Christensen [4] describes separation values that are calculated assuming that the required separation between two satellites is approximately constant regardless of the positions of the satellites on the orbital arc.

Wang's separation values were used in the test problems we present. As stated earlier, the primary goal in satellite synthesis models is to prevent excessive interference. The aggregate interference at each test point, i.e., the interference due to all unwanted satellite signals, is the quantity of concern; but the minimum pairwise separations recommended by Wang correspond to single-entry interference caused by one satellite at a time at each test point. In practice, it is found that the aggregate interference requirement can be satisfied by imposing a more stringent requirement, typically an additional 5 dB, on the single-entry interferences. As an example, suppose we require aggregate C/I ratios of at least 25 dB. Appropriate satellite separations might be calculated assuming a single-entry co-channel protection ratio of about 30 dB. Such a procedure was adopted for WARC-77 [5] and has proved valid in our test problems, as demonstrated below.

The significance of the minimum separation concept is that we avoid, in the optimization, the cumbersome expressions for interference that others such as Ito, Mizuno, and Muratani [7], Levis, Martin, Gonsalvez, and Wang [9], Martin et al. [11], and Reilly, Mount-Campbell, et al. [16] have used previously. In the place of these expressions, we use a conservative constant separation value for each pair of satellites, equal to the largest of the separation values calculated over the satellites' feasible orbital arcs, as recommended by Wang [18]. In effect, the problem has been separated into two parts: first, the calculation of separations based on interference

requirements, and then the optimization of the satellite orbit positions, subject to the separation constraints.

MIXED INTEGER PROGRAMMING FORMULATION

Using the minimum satellite separation values described above, we can formulate the FSS satellite system synthesis problem as a mixed integer program (MIP). A branch-and-bound algorithm can be used to solve this model for a global optimum. However, most integer programming problems are among the most computationally difficult optimization problems to solve. Solution times tend to increase dramatically with increases in the number of decision variables that are restricted to integer values; in fact, solution times may grow exponentially as the number of discrete-valued variables increases linearly. Therefore, in practice, the usefulness of this formulation for solving large synthesis problems to optimality is suspect. However, attractive feasible solutions are likely to be found by terminating the branch-and-bound solution procedure prematurely, for example, after a specified number of solutions is examined.

Before presenting the MIP formulation, we state our assumptions and define our parameters and decision variables.

Assumptions:

1. Easternmost and westernmost feasible locations are given for each satellite. Issues such as minimum elevation

angle and rain attenuation may be considered in setting these location limits.

2. Minimum required separations are known for all pairs of satellites. These may be established empirically, or they may have been calculated from the interference requirements with suitable frequency and polarization assumptions.
3. A desired location is specified for each satellite.
4. The objective is to minimize the sum of the absolute deviations of the satellites' prescribed locations from their desired locations.

Parameters:

e_j, w_j = easternmost or westernmost feasible location for satellite j in degrees west longitude.

$$e = \min_j \{e_j\}, \quad w = \max_j \{w_j\}.$$

d_j = desired location for satellite j in degrees west longitude.

Δs_{ij} = minimum required separation between satellites i and j in degrees longitude.

Decision Variables:

x_j = relative location (degrees west of e_j in degrees west longitude) of satellite j .

x_j^+, x_j^- = degrees west(+) or east(-) of its desired location
that satellite j is located.

x_{ij} = 1 if satellite i is located west of satellite j
0 otherwise

The FSS satellite synthesis problem can now be formulated as a mixed integer program as follows:

$$\text{Minimize } z = \sum_j (x_j^+ + x_j^-) \quad (1)$$

$$\text{Subject to } x_j - x_j^+ + x_j^- = d_j - e_j \quad \text{for all } j \quad (2)$$

$$x_i - x_j + (e - w - \Delta s_{ij}) x_{ij} > e - w - e_i + e_j \quad \text{for all } i, j \text{ where } i < j \quad (3)$$

$$-x_i + x_j - (e - w - \Delta s_{ij}) x_{ij} > \Delta s_{ij} + e_i - e_j \quad \text{for all } i, j \text{ where } i < j \quad (4)$$

$$x_j < w_j - e_j \quad \text{for all } j \quad (5)$$

$$x_j, x_j^+, x_j^- > 0 \quad \text{for all } j \quad (6)$$

$$x_{ij} \in \{0, 1\} \quad \text{for all } i, j \text{ where } i < j \quad (7)$$

The objective function (1) totals all of the absolute deviations of the prescribed satellite locations from the corresponding desired

locations. The deviations themselves are measured in the first set of constraints (2). Constraints of type (5) ensure that feasible locations, those that permit the illumination of each satellite's service area(s), are selected for all satellites. These constraints should be enforced as simple bounds rather than as explicit structural constraints. Nonnegativity of continuous variables and integrality of binary variables are enforced by constraint sets (6) and (7), respectively.

Constraints (3) and (4) require special explanation. For each pair of satellites i and j , exactly one of the constraints of types (3) and (4) will be redundant. The one that is redundant is determined by whether satellite i is located west of satellite j , or vice versa. The nonredundant constraint enforces the required separation between satellites i and j . (These constraints are an example of dichotomous or "either-or" constraints which are commonly used to model necessary logical relationships in integer programming applications.) If $x_{ij} = 1$, that is, if satellite i is located west of satellite j , (3) and (4) reduce to:

$$(x_i + e_i) - (x_j + e_j) > \Delta s_{ij} \quad (3a)$$

$$-(x_i + e_i) + (x_j + e_j) > e - w \quad (4a)$$

Constraint (4a) would be redundant in this case. The constant $(e-w)$ which appears in (4a) is the additive inverse of the length of the entire arc segment being considered; hence, (4a) can never be

violated. The required satellite separation would be enforced by constraint (3a). If $x_{ij} = 0$, the constants on the right-hand sides of (3a) and (4a) would be interchanged. Constraint (4a) would enforce the required satellite separation and (3a) would be redundant.

At most one of the variables x_j^+ and x_j^- should be positive at a solution. Fortunately, this is guaranteed by the simplex method for linear programming that would be used to solve linear programming subproblems during the execution of branch-and-bound procedures. To illustrate the meaning of x_j^+ and x_j^- , consider a satellite j , with a desired location of, say, $d_j = 75^\circ\text{W}$ and an easternmost feasible location of, say, $e_j = 65^\circ\text{W}$, the values of x_j , x_j^+ , and x_j^- that correspond to three different locations of satellite j are shown below:

Location	x_j	x_j^+	x_j^-
70°W	5	0	5
75°W	10	0	0
81°W	16	6	0

At optimal solutions, x_j^+ , which can never be negative, will be positive if and only if the location prescribed for satellite j is west of its desired location. Similarly, x_j^- , which can also never be negative, will be positive if and only if the optimal location for satellite j is east of its desired location. One may recognize that values other than those given above for x_j^+ and x_j^- are feasible. Consider the case when satellite j is located at 81° , then for

example, $x_j^+ = 13$ and $x_j^- = 7$ are also feasible, while not satisfying the condition that only one can be positive. However, these other sets of values for the variables will result in a less desirable objective function value. At an optimal solution, at least one of the variables x_j^+ and x_j^- will have value zero.

In this formulation, we have been able to include two of the objectives which were mentioned in the introduction. First of all, interference is kept in check through the enforcement of constraint sets (3) and (4). Secondly, the objective function (1) favors synthesis solutions in which the satellites collectively assume positions near their desired locations.

Solution times for mathematical programming models are dependent upon the number of constraints and the number of variables included in the model. Suppose we wish to solve a synthesis problem in which there are m satellites. In this case, our formulation will have m^2+m structural constraints (2), (3), and (4), $3m$ continuous variables, and $m(m-1)/2$ binary variables. For $m=100$, we will have 4950 binary variables. Without question, a synthesis problem of this magnitude would be difficult to solve unless an efficient special-purpose solution procedure were developed. One possible approach is to decompose a large problem into smaller problems that are considerably easier to solve; the solutions to the smaller problems would then be combined to yield a complete, but perhaps suboptimal, solution.

We present another model for the same synthesis problem in the next section. In this second model, we employ only continuous variables. While we are not certain to find a global optimum when we use this second formulation, we expect considerably shorter computing times, especially for synthesis problems with 100 or more satellites.

ALMOST LINEAR PROGRAMMING FORMULATION

Linear programming problems are among the most readily solvable optimization problems. Solution times tend to grow slowly as the number of decision variables is increased. Increases in the number of structural constraints typically produce polynomial increases in solution times. It would be a distinct advantage if we were able to formulate satellite synthesis problems as linear programs. Below, we describe a formulation which is nearly a linear program. The logical relationships which we enforced with binary variables in the formulation of the previous section can not be modeled exclusively with linear functions of continuous variables. If we are willing to model these logical relationships with nonlinear functions which we will enforce in a manner different from that used to enforce the remaining linear constraints, we may use the essential elements of the simplex method for linear programming to find solutions to the synthesis problem.

We first define new continuous variables for this formulation:

p_{ij} , n_{ij} = degrees west or east of satellite j that
satellite i is located

Our almost linear programming (ALP) formulation is:

$$\text{Minimize } z = \sum_j (x_j^+ + x_j^-) \quad (8)$$

$$\text{Subject to } x_j - x_j + x_j^+ = d_j - e_j \quad \text{for all } j \quad (9)$$

$$x_i - x_j - p_{ij} + n_{ij} = e_j - e_i \quad \text{for all } i, j \text{ where } i < j \quad (10)$$

$$p_{ij} + n_{ij} \geq \Delta s_{ij} \quad \text{for all } i, j \text{ where } i < j \quad (11)$$

$$x_j \leq w_j - e_j \quad \text{for all } j \quad (12)$$

$$x_j, x_j^+, x_j^- \geq 0 \quad \text{for all } j \quad (13)$$

$$p_{ij}, n_{ij} \geq 0 \quad \text{for all } i, j \text{ where } i < j \quad (14)$$

$$p_{ij} * n_{ij} = 0 \quad \text{for all } i, j \text{ where } i < j \quad (15)$$

The objective function (8) and the constraints of types (9), (12), and (13) are identical to, and serve the same purposes as, their counterparts in the mixed integer programming formulation. The

absolute distance between satellites i and j is measured by the sum of n_{ij} and p_{ij} in constraints (10). In the constraints of type (11), we guarantee that these distances are at least equal to the corresponding minimum separations. Nonnegativity restrictions on the new variables are enforced by constraints (14). The nonlinear constraints of type (15) serve an important purpose. A satellite can not be located both east and west of another satellite; it is necessary to include these constraints, as no others preclude a solution suggesting such a physical impossibility. These complementarity constraints require that at least one of the variables p_{ij} and n_{ij} must have value zero, for each pair of satellites.

Were it not for the complementarity constraints (15), this model could be solved using the simplex method for linear programming. With a slight modification in the simplex procedure, we have a heuristic algorithm to solve the problem formulated above. This modified algorithm is called the simplex method with restricted basis entry (RBE) [6,14]. The solutions found to satellite synthesis problems with this procedure may not be global optima.

This formulation has m^2+2m continuous variables and m^2+m constraints, where m is the number of satellites. Although this model is really no smaller than the mixed integer programming model, it is expected that substantially less computing time will be needed to find solutions, especially when the number of satellites is large. The absence of discrete-valued variables may be a significant advantage,

particularly when solving problems with a large value of m . Furthermore, heuristic approaches, such as the simplex method with RBE in this case, tend to find better approximate solutions as problem size increases.

In the next section, we suggest additional synthesis formulations which consider the minimization of the length of the orbital arc occupied by the satellites we seek to position.

MINIMUM ORBITAL ARC UTILIZATION AS AN OBJECTIVE

The objective of locating satellites as near as possible to specified desired locations is a reasonable goal. However, it is not the only viable objective. The geostationary orbit is a limited natural resource, and it is likely that the demand for satellite positions in this orbit will continue to increase. In order to accommodate requests for additional satellite positions in the future, a reasonable alternative objective would be to minimize the length of the orbital arc occupied by the satellites to be positioned. With some modification, our mathematical programming formulations can accommodate this new objective. Assumptions (3) and (4), from our earlier formulations, are now relaxed in the formulations that seek to minimize the length of the occupied orbital arc.

Two new variables are needed for the formulations that consider the utilization of the orbital arc:

x_0 = dummy satellite location at the eastern edge of the occupied arc, equal to the location of the easternmost satellite.

x_{m+1} = dummy satellite location at the western edge of the occupied arc, equal to the location of the westernmost satellite.

The new MIP formulation is:

$$\text{Minimize } z = x_{m+1} - x_0 \quad (16)$$

Subject to

$$x_i - x_j + (e - w - \Delta s_{ij}) x_{ij} > e - w - e_i + e_j \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (17)$$

$$-x_i + x_j + (e - w - \Delta s_{ij}) x_{ij} > \Delta s_{ij} + e_i - e_j \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (18)$$

$$x_0 - x_j < e_j \quad \text{for } j=1, 2, \dots, m \quad (19)$$

$$x_{m+1} - x_j > e_j \quad \text{for } j=1, 2, \dots, m \quad (20)$$

$$x_j < w_j - e_j \quad \text{for } j=1, 2, \dots, m \quad (21)$$

$$x_j > 0 \quad \text{for } j=0, 1, \dots, m+1 \quad (22)$$

$$x_{ij} \in \{0, 1\} \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (23)$$

The objective function (16) seeks to minimize the distance between the dummy satellite locations x_0 and x_{m+1} , which is the length of the arc occupied by the satellites to be positioned. Constraint types (17) and (18) enforce the minimum satellite separation requirements to maintain interference at an acceptable level. Constraint sets (19) and (20) force x_0 and x_{m+1} to assume values which differ by at least as much as the greatest separation between any two satellites. The locations prescribed for the satellites are restricted to feasible portions of the orbital arc by constraint set (21). The remaining constraints (22) and (23) ensure the nonnegativity of the continuous variables and the integrality of the integer variables.

The analogous ALP formulation is:

$$\text{Minimize } z = x_{m+1} - x_0 \quad (24)$$

Subject to

$$x_i - x_j - p_{ij} + n_{ij} = e_j - e_i \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (25)$$

$$p_{ij} + n_{ij} \geq \Delta s_{ij} \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (26)$$

$$x_0 - x_j \leq e_j \quad \text{for } j=1, 2, \dots, m \quad (27)$$

$$x_{m+1} - x_j \geq e_j \quad \text{for } j=1, 2, \dots, m \quad (28)$$

$$x_j \leq w_j - e_j \quad \text{for } j=1,2,\dots,m \quad (29)$$

$$x_j \geq 0 \quad \text{for } j=0,1,\dots,m+1 \quad (30)$$

$$p_{ij}, n_{ij} \geq 0 \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (31)$$

$$p_{ij} * n_{ij} = 0 \quad \begin{array}{l} \text{for all } 1 \leq i, j \leq m \\ \text{where } i < j \end{array} \quad (32)$$

The objective function (24) is the same as that in the previous model. The actual separations between all pairs of satellites are determined in constraint set (25). Constraints of type (26) enforce required minimum separations between all pairs of satellites. Constraint sets (27) and (28) guarantee that x_0 and x_{m+1} will differ by at least as much as the greatest separation between any two satellites. Feasible locations for the satellites are ensured by constraints (29). Nonnegativity and complementarity restrictions on the decision variables are enforced by constraints (30), (31), and (32).

For a synthesis problem with m satellites, the MIP would have m^2+2m constraints, $m+2$ continuous variables, and $m(m-1)/2$ discrete variables. There would be m^2+2m constraints and m^2+2 variables in the ALP.

These new models may be solved via the same methods as the models presented earlier. The large number of discrete-valued variables in the MIP means that finding a solution will require a lengthy effort. The ALP is expected to require less computing time, but there is no guarantee that a global optimal solution will be found.

OTHER POSSIBLE FORMULATIONS

Our primary purpose in this report is to suggest alternative formulations of satellite system synthesis problems. Thus far, two different objective functions have been considered in our formulations. These objectives do not constitute an exhaustive list of reasonable objectives. Given a set of minimum satellite separations that ensure satisfactory protection from interference, a variety of additional MIP's and ALP's might be formulated.

Some examples of additional objectives that can be used along with the minimum satellite separation concept in mathematical programming formulations of satellite synthesis problems are:

1. Minimization of a weighted sum of the deviations of assigned satellite locations from corresponding pre-assigned desired locations.
2. Minimization of the largest deviation of any satellite's assigned location from its desired location.

3. Maximization of the smallest separation between any pair of adjacent satellites.
4. Minimization of a weighted combination of arc length and absolute deviations between the satellites' desired and assigned locations.
5. Maximization of the minimum amount, among all satellite pairs, by which an actual separation exceeds the minimum required separation.

The synthesis solutions found for some of these objectives may exhibit special desirable properties. For example, it is expected that objectives (2), (3), and (5) would force the performance measures for the individual satellites, for example, the deviation of an assigned location from the corresponding desired location for additional objective 2, to have similar values, with no regard for the total of the satellites' performance measures. An optimal solution would then be about equally attractive for all administrations. Thus, no administration would be penalized in a disproportionate manner by implementation of the solution found.

SOLUTIONS TO TEST PROBLEMS

In this section, we present the solutions obtained for four synthesis test problems, using the MIP and ALP formulations with the

objective of positioning the satellites as near as possible to desired locations. Six South American administrations - Argentina (ARG), Bolivia (BOL), Chile (CHL), Paraguay (PRG), Peru (PRU), and Uruguay (URG) - each with one satellite, are included in the first three test problems. A single-entry protection ratio of 30 dB is assumed, with the intent of achieving aggregate C/I ratios of at least 25 dB. These test problems and solutions are the same as those presented by Levis, Wang, et al. [10]. However, our MIP formulation is different than the one used by Levis, Wang, et al. We have fewer continuous variables and constraints in our MIP formulation. The solution times we report for our MIP formulation are therefore more favorable. The MIP problems were solved with a branch-and-bound code; the simplex method with RBE was used to solve the ALP problems. All computer runs were made on an IBM 3081-D computer at The Ohio State University.

The minimum satellite separations used for the first three test problems are displayed in Table 1. All matrix entries are in degrees of orbital arc.

Table 1. Minimum Satellite Separations

	BOL	CHL	PRG	PRU	URG
ARG	4.17	4.19	4.32	1.41	4.14
BOL		4.57	4.04	4.26	0.94
CHL			2.00	3.94	1.59
PRG				1.10	2.46
PRU					0.37

We assume that all six satellites can occupy any location between 80°W and 110°W. Target locations, the MIP solution, and the ALP solution for three versions of this test problem are summarized below in Tables 2, 3, and 4. All entries in these tables are in degrees west longitude.

Table 2. Solutions to Test Problem 1

Satellite	Satellite Locations		
	Desired	MIP	ALP
ARG	95.0	88.68	105.74
BOL	95.0	99.57	101.57
CHL	95.0	95.00	97.00
PRG	95.0	93.00	95.00
PRU	95.0	91.06	93.06
URG	95.0	96.59	92.54
Obj. fn. value		18.42	23.71
CPU sec.		4.01	1.31

Table 3. Solutions to Test Problem 2

Satellite	Satellite Locations		
	Desired	MIP	ALP
ARG	110.0	101.35	110.00
BOL	110.0	97.18	104.33
CHL	110.0	105.54	99.76
PRG	110.0	107.54	97.76
PRU	110.0	109.63	108.59
URG	110.0	110.00	105.86
Obj. fn. value		28.76	33.69
CPU sec.		3.24	1.30

Table 4. Solutions to Test Problem 3

Satellite	Satellite Locations		
	Desired	MIP	ALP
ARG	87.5	88.76	101.26
BOL	92.5	92.93	92.50
CHL	97.5	97.50	97.07
PRG	87.5	84.44	87.50
PRU	102.5	102.50	102.67
URG	82.5	81.98	82.50
Obj. fn. value		5.27	14.36
CPU sec.		0.69	1.25

Six different synthesis solutions were found. The MIP model produced better solutions than the ALP model, as measured by objective function values. All six synthesis solutions found keep interference at an acceptable level. Levis, Wang, et al. [10] report that the aggregate co-channel C/I ratios computed at 54 test points located on the perimeters of the six administrations are all above 27 dB for all six solutions found. This satisfies the goal that the aggregate C/I ratios for all channels combined should be 25 dB or more. It appears, then, that these satellite separations have indeed served their purpose.

The solution times for the six examples solved are not excessive. We find that there are almost no differences in the times required to solve the ALP examples. Solution times for the MIP examples exhibit much more variability. As larger synthesis problems are solved, the solution times for both models will increase. Mixed integer programming solution times are likely to grow much faster and vary much more than those for the almost linear programs.

A second test problem, with 13 satellites serving South American administrations, has also been solved using the mixed integer and almost linear programming formulations and the corresponding solution techniques. For this test problem, a contrived, but geographically consistent, matrix of required minimum separation values was constructed. The purpose was not to develop a practical scenario for these administrations, but to examine the growth in solution time and

the behavior of the objective function as the number of satellites is increased. The feasible arc for each satellite was assumed to be 60°W to 100°W. A single-entry co-channel protection ratio of 30 dB was used for this test problem. The desired locations and solutions found, in degrees west longitude, are summarized in Table 5.

Solution times for both approaches are greater for this test problem. This result is expected as the number of satellites has been increased. The increase in solution time for the MIP model is substantially greater than that for the ALP model, as expected. However, the objective function value for the MIP solution is significantly more attractive than that for the ALP solution. It is hoped that discrepancies as large as this one will be atypical, especially for problems with many satellites.

Table 5. Thirteen Satellite Test Problem and Solutions

	Desired	MIP	ALP
Service Area	Location	Solution	Solution
Brazil-1	60.00	60.00	60.00
Surinam/ Fr. Guiana	65.00	65.00	64.62
Guyana	67.50	69.07	68.69
Paraguay	67.50	65.04	68.64
Uruguay	67.50	67.50	66.18
Argentina	70.00	71.64	87.38
Brazil-2	75.00	80.42	73.20
Venezuela	75.00	75.00	77.62
Bolivia	77.50	75.81	83.21
Chile	85.00	85.00	91.57
Colombia	85.00	84.58	91.15
Peru	87.50	88.94	78.95
Ecuador	90.00	92.95	86.87
Obj. fn. value		17.59	55.94
CPU sec.		34.17	8.40

CONCLUSIONS

The literature shows that there are many reasonable formulations of satellite system synthesis problems. A variety of objectives have been used. Some formulations may be preferable to others for computational reasons, or because a particular formulation optimizes a particular objective or combination of objectives.

All of the formulations we have suggested here are made possible by applying a minimum satellite separation concept. They enforce single-entry interference objectives while attempting to find a synthesis solution which optimizes an additional criterion. Cumbersome nonlinear expressions for interference relationships are avoided in the optimization. Rather, we use a set of constants that facilitate the use of linear functions of decision variables in our formulations. These constants are minimum pairwise satellite spacings that enforce the interference requirements. In the approach adopted here for numerical examples, the constants are pre-calculated from single-entry interference constraints; this is more efficient computationally than the repeated interference calculations made in nonlinear programming approaches [7,9,15].

As part of our ongoing research effort, we intend to investigate the performance of the branch-and-bound method and the simplex method with RBE on the formulations we have recommended here for larger synthesis examples. We also hope to characterize the solutions obtainable with different objectives.

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